

A MATHEMATICAL MODELLING FOR SHIP STABILITY

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ABSTRACT

This paper work describes the modeling of a stability of ship. Floating bodies subject to the disturbances such as external force, gravity force and etc. The stability depends on the location of the body, weight, ship structure. The concept of ship stability can be found in terms of applications of buoyancy force, centre of gravity, moments. An approach using the concept of finding equilibrium steady states corresponding to the radii value is presented. Some numerical examples are given to illustrate the effectiveness of the presented work.

KEYWORDS: Ship Stability, Buoyancy, Floating bodies, Mathematical Modeling.

INTRODUCTION:

The concept of ship stability is one of the most significant areas of centre of attention in ship design and operation, not only to guarantee the safety of the ship, cargo, crew and passengers, but also to enable proper conditions for completion of all the processes on a ship.

Also, some concepts are applied in real-time and probable situations to analyze the stability of the ship, how a designer applies concepts of hydrostatics and stability to develop a hull form, and so on.

Ship stability is certainly a subject of paramount significance in the field of Naval Architecture, the design and operation of ships and floating units having a fundamental role. Moreover, “stability” is a notion which has a very broad significance in Naval Architecture, especially embracing ship stability fundamentals with ship dynamics and eventually safety of the ship. In this admiration, research in the field has arrived considerable attention within the whole maritime community, ensuing in the modern-day evolution of the subject to the integrated idea of “ship stability, dynamics and safety” as it is being presently acceptable. This technical paper concludes with a method that helps to find the metacentric height in order to analyses the stability of the ship. In this way, a very few prevailing principles are followed by the naval engineer, who is able to determine which of the existing criteria fits best with the requirements of the ships function.

The statically stability of ships is checked by comparing the righting-arm curve with the curves of heeling arms. A heeling arm is calculated by dividing a heeling moment by the ship displacement force. The concept of ship stability can be found in terms of applications of buoyancy force, center of gravity, moments. An approach using the concept of finding equilibrium steady states corresponding to the radii value is presented.

This paper is organized as follows. Some basic definitions and theoretical concepts have been discussed. A study has been done on the ship stability by using the prevailing concepts. Finally some numerical examples were given to illustrate the theory.

PRELIMINARIES:

BUOYANCY FORCE:

The capacity of an object to float is simply defined as a term “Buoyancy”. When any object is placed in any fluid, the fluid exerts an upward force this upward force is known as Buoyancy Force.

This is because of the pressure exerted on the object by the fluid. It is the force on the body which allows it to float or sink when it is placed in fluid

ARCHIMEDES' PRINCIPLE

A body in liquid experience a Buoyant upward force equal to the liquid displaced by that body.

BUOYANT FORCE EQUATION:

$$\text{Buoyant force } F_{\text{buoyant}} = \rho_{\text{fluid}} g V$$

where,

ρ = density of the fluid

g = gravitational acceleration

V = Volume of the fluid displaced

Since, $V = m/\rho_{\text{object}}$

Buoyant force can be expressed as

$$F_B = m g \rho_{\text{fluid}} / \rho_{\text{object}}$$

STABILITY:

Stability is the property, quality, or characteristic of a body, which cause it, when its equilibrium is disturbed, to develop forces or moments acting to restore its original condition

SHIP STABILITY:

The surface ship's stability can be divided into two parts,

- Intact Stability
- Damaged Stability

INTACT STABILITY:

The stability of a surface ship when the intactness of its hull is maintained, and no compartment or watertight tank is damaged or freely flooded by seawater.

DAMAGED STABILITY

The stability of a surface ship includes the identification of compartments or tanks that are subjected to damage and flooded by sea water, followed by a prediction of resulting trim and draft conditions.

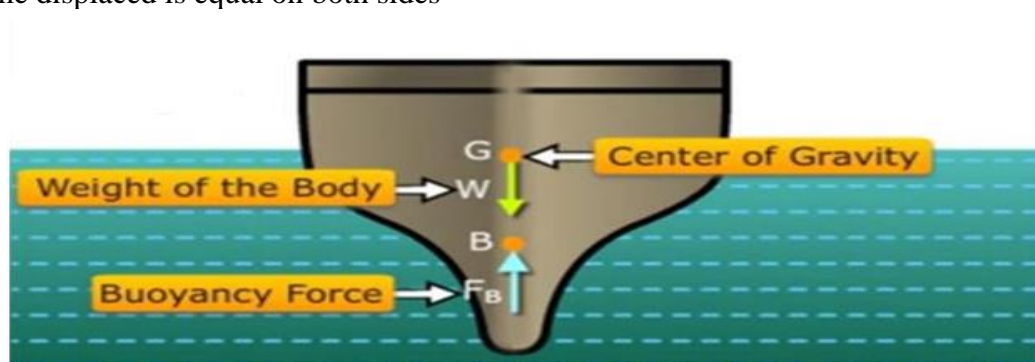
STABILITY OF SHIP:

Ship stability deals with how a ship behaves at sea, both in still water and in waves. Stability calculations focus on the centre of gravity, centre of buoyancy, and metacentre of the ship.

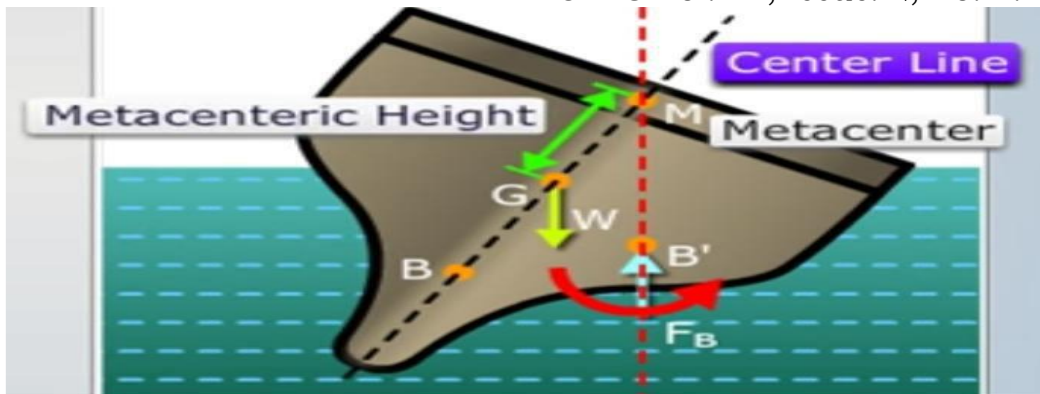
Stability of equilibrium of a Floating body depends, upon the location of the metacentre. If a ship floats, mass density of the ship is less than that of the water. Weight of the ship is same as the water displaced by the ship.

To be in equilibrium,

- ❖ Buoyant force must be equal to the weight of the ship displaced by the water.
- ❖ Gravitational force and buoyant force should lie on the same vertical line.
- ❖ Volume displaced is equal on both sides



When a body undergoes an angular displacement towards right side, then the volume displaced is larger in right side and lesser in left side. Thus the centre of Buoyancy shifts to the right from B to B' and a couple is formed which tries to rotate the body. Here the point of intersection of the vertical line through the new centre of buoyancy B' and the centre line G is turned as **Metacentre** and the distance by which the metacentre lies above the centre is gravity is known as **Metacentric height**
 Metacentric height (GM) = BM-BG



TYPES OF EQUILIBRIUM:

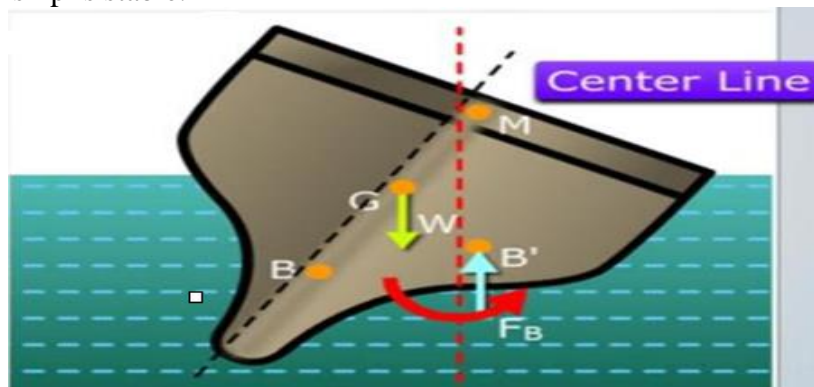
The concept behind the understanding of intact stability of a floating body is that of Equilibrium. There are three types of equilibrium conditions that can occur, for a floating ship, depending on the relation between the positions of center of gravity and center of buoyancy.

1. Stable Equilibrium
2. Unstable Equilibrium
3. Neutral Equilibrium

STABLE EQUILIBRIUM:

When M is above G, then the restoring couple is formed by the buoyant force and the weight of the body which tends to change the body to original position.

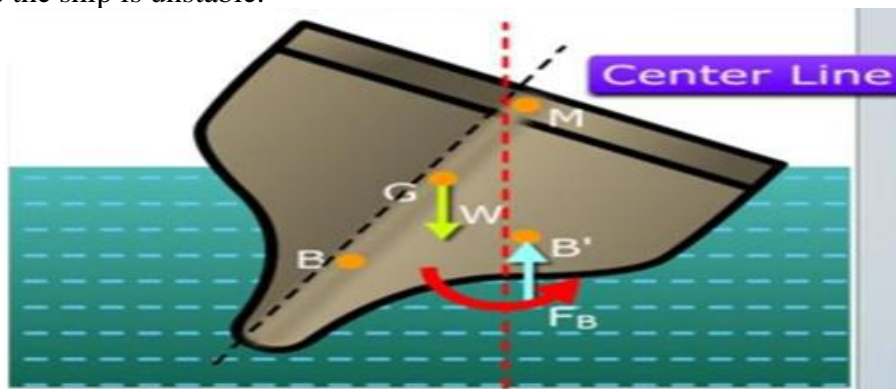
$GM > 0$ means the ship is stable.



UNSTABLE EQUILIBRIUM:

When M is below G, Then overturning couple is formed by the Buoyancy force and the weight of the body which tends to sink the body from its original position.

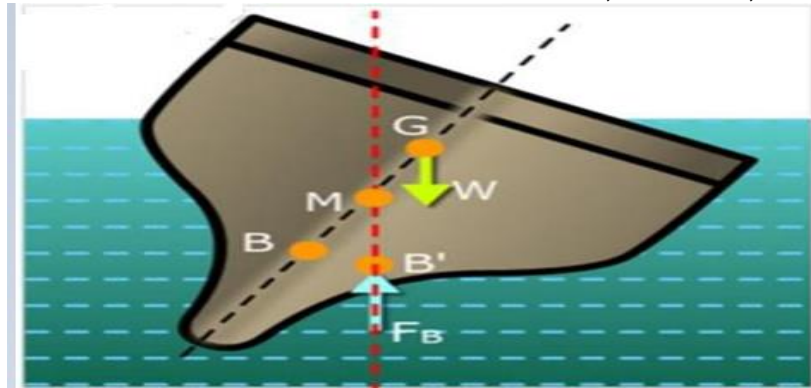
$GM < 0$ means the ship is unstable.



NEUTRAL EQUILIBRIUM:

When M coincides with G, Then line of action of buoyant force and weight of the body are collinear and passes through same point due to this the ship neither returns to its original position nor increases its displacement.

$GM = 0$ means the ship is neutrally stable.



THE STABILITY OF PARTIALLY SUBMERGED BODIES:

Let G be the centre of gravity

B be the centre of buoyancy

OO be the original water surface

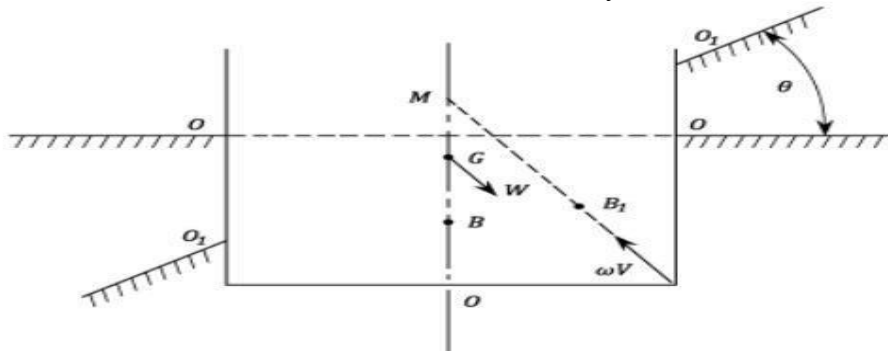
O₁ O₁ be the new water line

θ be the angle of tilt

GM be the metacentric height

G remains in the same position but B moves to B₁

Righting couple = $W GM \sin\theta$ where W is the mass of the body.



THE DETERMINATION OF METACENTRIC HEIGHT:

Let W be the weight of the ship plus its load. A small load w is moved to a distance of x and causes a tilt of angle θ . The ship is now in a new position of equilibrium with B' and G' lying along the vertical through M.

The moment due to Centre of gravity is

$$w_x = W * GM \theta$$

THEORETICALLY:

The ship tilts from its original water OO line to new water line O'O' and it moves to an angle θ .

Due to the movement of the wedge of water from A₁AC and D₁DC the center of Buoyancy moves from B to B'.

$$GM = w_x / w\theta$$

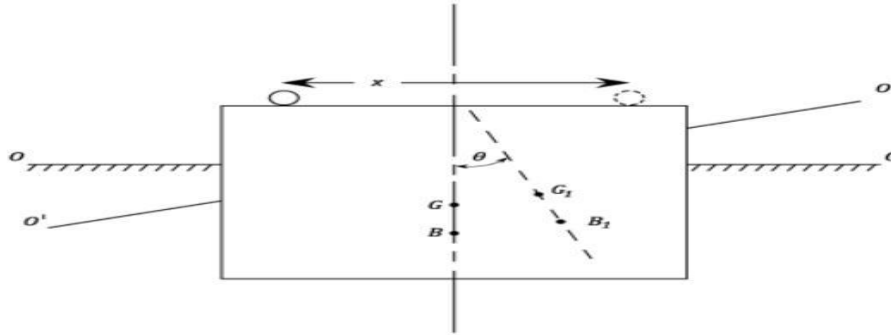
The volume of the wedge is ACA'

$$= \left(\frac{1}{2}\right) \left(\frac{b}{2}\right) \left(\frac{b}{2}\right) \theta \times L = \frac{2bL\theta}{8}$$

The Moment of the Couple due to the movement of the wedge = $\left(\frac{w2bL\theta}{8}\right) \left(\frac{2b}{3}\right)$

The Change in the moment of the buoyancy Force = $wV \times BB'$

$$= wV \times BM\theta \text{ where } \theta \text{ is small}$$



V is the volume of water displaced

If the values of G & B are known GM can be determined.

$$GM = BM + BG$$

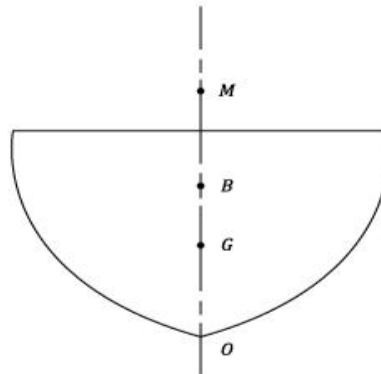
$$= BM + BO - OG$$

$$wV \times BM\theta = \left(\frac{w3bL\theta}{12} \right)$$

$$BM = \left(\frac{b3L}{12V} \right)$$

$$= \left(\frac{I}{L} \right)$$

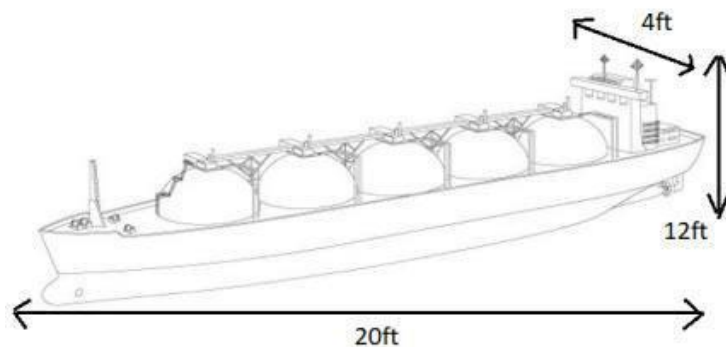
Where I is the Second Moment of Area of the Water Plane Section



APPLICATION PROBLEMS:

PROBLEM 1:

A ship measuring 20ft. by 12ft. and 4ft deep, weighs 12 tons. It carries a load of 8 tons. The ship is in sea water with a density of 64 lb/cu. ft. Find its metacentric height and establish the angular tilt which will result if the load is moved by one ft. sideways.



SOLUTION:

$$\begin{aligned} \text{Moments above the base} &= (12 + 8) \times OG \\ &= 12 \times OG_P + OG_L \end{aligned}$$

$$OG = 12 \times 2 + 8 \times \frac{5}{(12+8)}$$

$$= 3.2 \text{ ft.}$$

$$\begin{aligned}\text{The volume of water displaced } V &= (12+8) \times \frac{2240}{64} \\ &= 700\text{ft}^3\end{aligned}$$

$$\begin{aligned}\text{Depth of immersion } h &= \frac{700}{20} \times 12 \\ &= 2.197 \text{ ft.}\end{aligned}$$

$$\text{The height } OB = \frac{h}{2} = 1.458\text{ft}$$

$$\begin{aligned}BM &= \frac{Lb_3}{12V} \\ &= \frac{(20 \times 12^3)}{(20 \times 700)} \\ &= 4.143\text{ft}\end{aligned}$$

$$\begin{aligned}\text{Metacentric height} &= BM + OB - OG \\ &= 4.143 + 1.458 - 3.2 \\ &= 2.401\text{ft}\end{aligned}$$

The moment due to the Movement of the Load = 8 ft tones

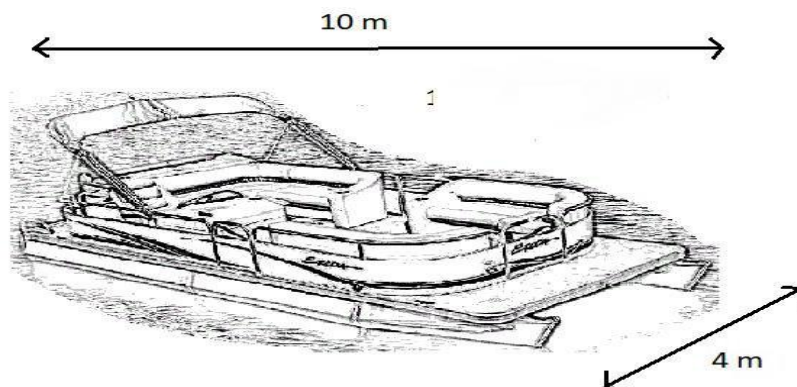
The moment due to the movement of the center of gravity = $W \theta = 20 \times 2.40$

$$\theta = \frac{8}{48} = \frac{1}{6} \text{ radian} = 9^\circ 31'$$

The metacentric height is 2.40ft and the angle is θ is $9^\circ 31'$

PROBLEM 2:

A boat 10m by 4m in plan, weighs 280 kN and floats in sea water of density 1025 kgm⁻³. A steel tube weighing 34 kN is placed longitudinally on the deck. When the tube is in a central position, the center of gravity for the combined mass is on the vertical axis of symmetry 0.25m above the water surface. Find the metacentric height?



SOLUTION:

$$\begin{aligned}\text{Weight of boat + load} &= 280 + 34 \\ &= 314 \text{ kN}\end{aligned}$$

$$\text{Weight of seawater displaced} = 1025 \times 9.8 \times 10 \times 4 \times \text{Draught}$$

$$\begin{aligned}\text{Draught} &= \frac{(314 \times 1000)}{(1025 \times 9.8 \times 10 \times 4)} \\ &= 0.781\text{m}\end{aligned}$$

$$\begin{aligned}BM &= \frac{I}{V} \\ &= \frac{(1/12 \times 10 \times 4^3)}{4 \times 10 \times 0.781} \\ &= 1.707\text{m}\end{aligned}$$

$$\begin{aligned}\text{The center of gravity is } BG &= \frac{0.25 + 0.781}{2} \\ &= 0.640\text{m}\end{aligned}$$

$$\begin{aligned}GM &= BM - BG \\ &= 1.707 - 0.640 \\ &= 1.067\text{m}\end{aligned}$$

The metacentric height is 1.067m

CONCLUSION:

From our analysis the stability of the ship depends on the metacentre and the centre of buoyancy plays an important role in Metacentre. When metacentre is equal to zero the ship remains in neutral equilibrium and when it is less than or greater than the ship remains unstable and stable respectively.

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